Controlling Large-Scale Systems with Distributed Model Predictive Control

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Outline

1. Overview of Distributed Model Predictive Control
   - Control of large-scale systems

2. Cooperative Control
   - Stability theory for cooperative MPC

3. Conclusions and Future Outlook

4. Some Comments on Tom Edgar
What are the goals of MPC?

- Choose inputs which bring outputs to their setpoints
- Minimize objective function over $N$ future steps

Minimize objective function:

$$
\min_{u} \quad V(x, u) = \sum_{k=0}^{N-1} \ell(x(k), u(k)) + V_f(x(N))
$$

subject to:

$$
x^+ = Ax + Bu \\
y = Cx
$$
Chemical plant integration

Material flow

Energy flow
Ideal plantwide MPC

\[ \min_{u_1, u_2} V(u_1, u_2) \]

- Ideal controller
  - perfect model
  - never goes offline
  - optimizes infinitely fast
  - samples infinitely fast
Practical plantwide MPC

\[
\begin{align*}
\min_{u_1} & \quad \tilde{V}_1(u_1) \\
\min_{u_2} & \quad \tilde{V}_2(u_2)
\end{align*}
\]

- Realistic controller
  - approximate model
  - MPCs fail or require maintenance
  - finite optimization time
  - multiple sampling rates

- Goal: make realistic controller close to ideal controller
Plantwide distributed MPC

MPC 1
\[ \min_{u_1} V(u_1, u_2) \]

MPC 2
\[ \min_{u_2} V(u_1, u_2) \]

- Decentralized control
  - no communication
  - not stable for strongly interacting subsystems

- Noncooperative control
  - use full modeling information
  - not stable for strongly interacting subsystems

- Cooperative control
  - use same objective in each controller
  - stability independent of interaction strength
Cooperative model predictive control

\[ V(u_1, u_2) \]

\[ u_0, u_1, u_2, u_0^* \]

MPC 1

MPC 2
Plantwide suboptimal MPC

Early termination of optimization gives suboptimal plantwide feedback

Use suboptimal MPC theory to prove stability
Plantwide suboptimal MPC

Consider closed-loop system augmented with input trajectory

\[
\begin{pmatrix}
    x^+ \\
    u^+
\end{pmatrix} = \begin{pmatrix}
    A & B \\
    g(x, u)
\end{pmatrix}
\]

- Function \( g(\cdot) \) returns suboptimal choice
- Stability of augmented system is established by Lyapunov function

\[
0 \leq V(x, u) \leq b |(x, u)|^2
\]

\[
V(x^+, u^+) - V(x, u) \leq -c |(x, u)|^2
\]

- Adding constraint establishes closed-loop stability of the origin for all \( u \)

\[
|u| \leq d |x| \quad x \in \mathbb{B}_r, r > 0
\]

- Cooperative optimization satisfies these properties for plantwide objective function \( V(x, u) \)

\(^1\) (Rawlings and Mayne, 2009, pp.418-420)
LV control of distillation column

- MPC 1
- MPC 2

- $S$
- $R$
- $X_D$
- $X_B$
LV control of distillation column

![Graphs showing control performance over time for distillation column](image-url)
## Performance comparison

<table>
<thead>
<tr>
<th></th>
<th>Cost</th>
<th>Performance loss (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centralized MPC</td>
<td>75.8</td>
<td>0</td>
</tr>
<tr>
<td>Cooperative MPC (10 iterates)</td>
<td>76.1</td>
<td>0.388</td>
</tr>
<tr>
<td>Cooperative MPC (1 iterate)</td>
<td>87.5</td>
<td>15.4</td>
</tr>
<tr>
<td>Noncooperative MPC</td>
<td>382</td>
<td>404</td>
</tr>
<tr>
<td>Decentralized MPC</td>
<td>364</td>
<td>380</td>
</tr>
</tbody>
</table>
Plantwide topology

- Plant subsystems can often be grouped spatially or dynamically.
- Neighborhoods of subsystems naturally arise from topology.
Traditional hierarchical MPC

- Multiple dynamical time scales in plant
- Data and setpoints are exchanged on chosen scale
- Optimization performed at each layer

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2Mesarović et al. (1970); Scattolini (2009)
Cooperative MPC data exchange

- All data exchanged plantwide
- Data exchange at each controller execution

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Venkat (2006); Stewart et al. (2010b)
Cooperative hierarchical MPC

- Optimization at MPC layer only
- Only subset of data exchanged plantwide
- Data exchanged at chosen time scale

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4Stewart et al. (2010a)
The challenge of nonlinear models

\[ V(u_1, u_2) \]
Distributed gradient projection - example

\[ V(u_1, u_2) = e^{-2u_1} - 2e^{-u_1} + e^{-2u_2} - 2e^{-u_2} + 1.1 \exp(-0.4((u_1 + 0.2)^2 + (u_2 + 0.2)^2)) \]
Conclusions

Cooperative MPC theory maturing (Stewart et al., 2010b; Maestre et al., 2010)

- Satisfies hard input constraints
- Provides nominal stability for plants with even strongly interacting subsystems
- Retains closed-loop stability for early iteration termination
- Converges to Pareto optimal control in the limit of iteration
- Remains stable under perturbation from stable state estimator
- Avoids coordination layer
Future Outlook

Extensions required for practical implementation

- Can we treat nonlinear plant models? Qualified yes.
- Can we avoid coupled constraints? Qualified yes.
- Can we reduce the assumed complete communication? Yes.
- Can we accommodate time-scale separation? Yes.
- Can we nest layers within layers? Yes.
Personal observations of Tom

- He’s old
  I took optimization from Tom when I was an undergraduate!
  I did research with Tom when I was an undergraduate!
  My son is now doing research with Tom as an undergraduate!

- He’s wily
  I’ve played a lot of golf with Tom over the years at research meetings, and I want to make it clear he cheats!
  Tom was department chair when I joined the University of Texas as an assistant professor. He was a great mentor.

- He’s a lot of fun to be around
Further reading


