Controlling Large-Scale Systems with Distributed Model Predictive Control

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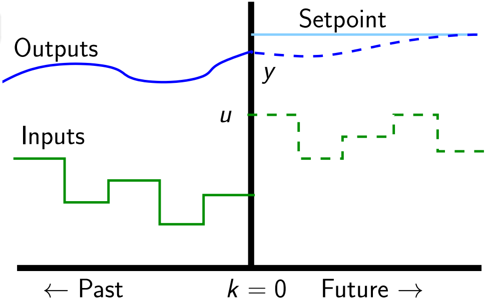
November 8, 2010
Annual AIChE Meeting
Salt Lake City, UT

Outline

Model predictive control

What are the goals of MPC?

- Choose inputs which bring outputs to their setpoints
- Minimize objective function over N future steps



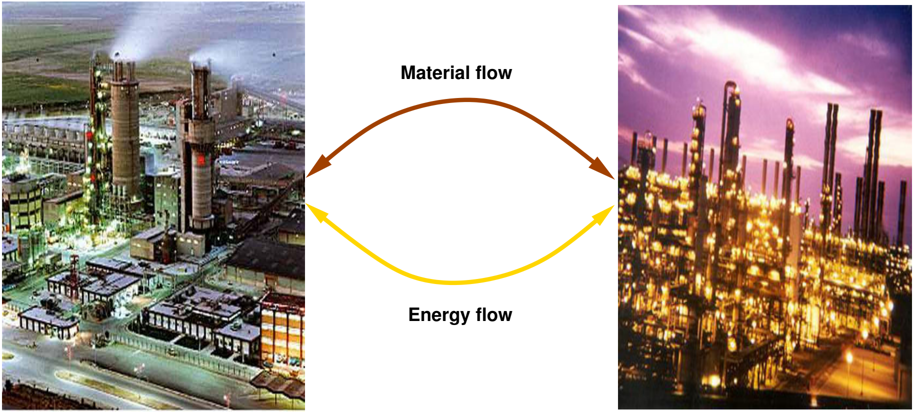
$$\min_{\mathbf{u}} V(x, \mathbf{u}) = \sum_{k=0}^{N-1} \ell(x(k), u(k)) + V_f(x(N))$$

subject to

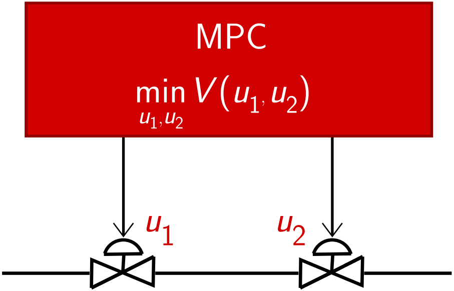
$$x^+ = Ax + Bu$$

$$y = Cx$$

Chemical plant integration

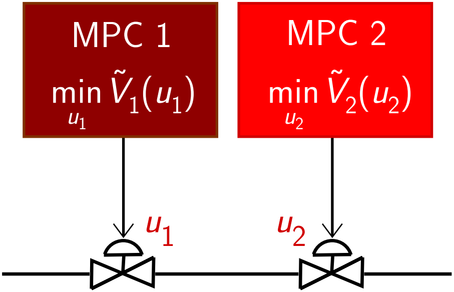


Ideal plantwide MPC



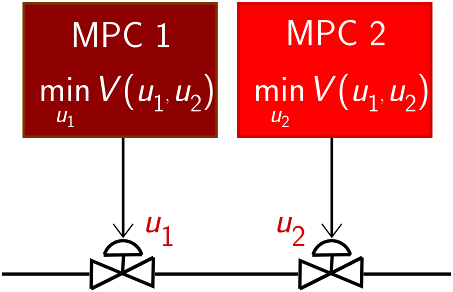
- Ideal controller
  - perfect model
  - never goes offline
  - optimizes infinitely fast
  - samples infinitely fast

Practical plantwide MPC



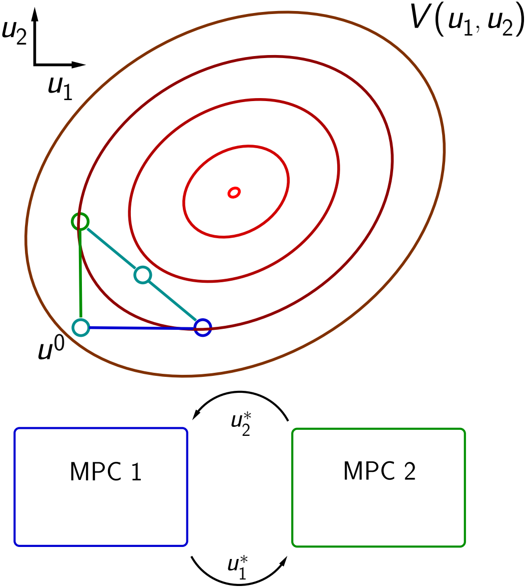
- Realistic controller
  - approximate model
  - MPCs fail or require maintenance
  - finite optimization time
  - multiple sampling rates
- Goal: make realistic controller close to ideal controller

Plantwide distributed MPC

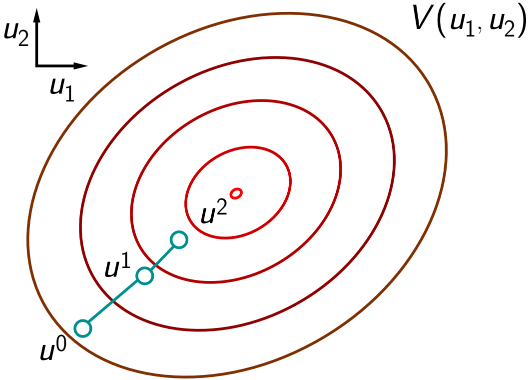


- Decentralized control
  - no communication
  - not stable for strongly interacting subsystems
- Noncooperative control
  - use full modeling information
  - not stable for strongly interacting subsystems
- Cooperative control
  - use same objective in each controller
  - stability independent of interaction strength

Cooperative model predictive control



Plantwide suboptimal MPC



- Early termination of optimization gives suboptimal plantwide feedback
- Use suboptimal MPC theory to prove stability

Plantwide suboptimal MPC

Consider closed-loop system augmented with input trajectory

$$\begin{pmatrix} x^+ \\ \mathbf{u}^+ \end{pmatrix} = \begin{pmatrix} Ax + Bu \\ g(x, \mathbf{u}) \end{pmatrix}$$

- Function $g(\cdot)$ returns suboptimal choice
- Stability of augmented system is established by Lyapunov function

$$a\left|(x,\mathbf{u})\right|^2 \leq V(x,\mathbf{u}) \leq b\left|(x,\mathbf{u})\right|^2$$
$$V(x^+,\mathbf{u}^+) - V(x,\mathbf{u}) \leq -c\left|(x,u)\right|^2$$

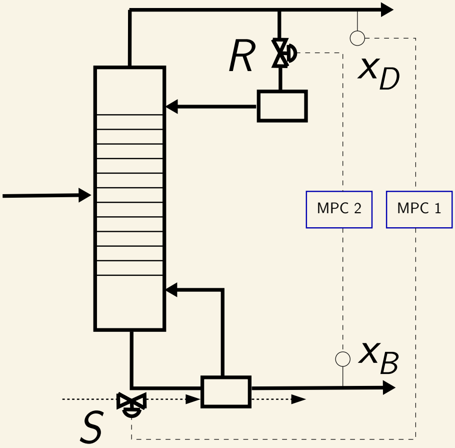
- Adding constraint establishes closed-loop stability of the origin for all $\mathbf{u}$[1]

$$|\mathbf{u}| \leq d\,|x| \quad x \in \mathbb{B}_r, r > 0$$

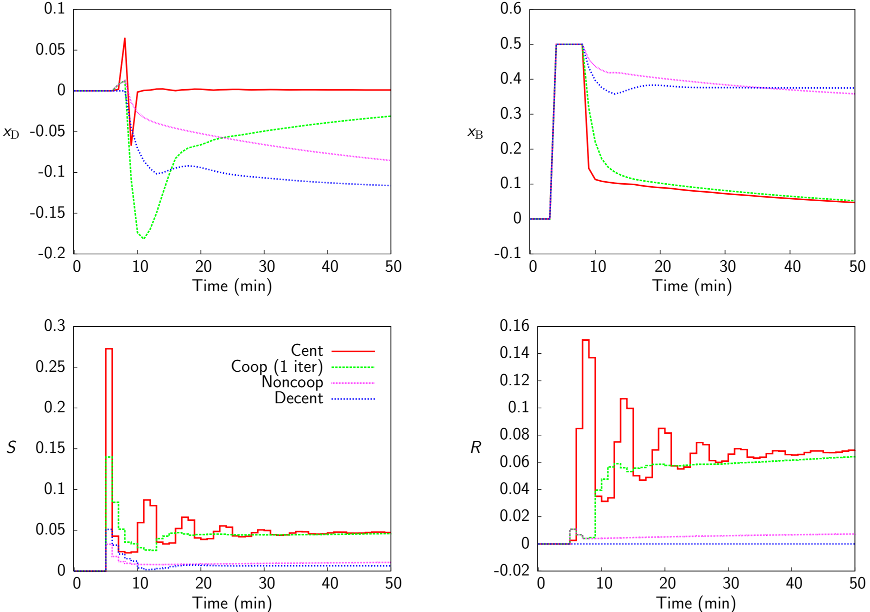
- Cooperative optimization satisfies these properties for plantwide objective function $V(x, \mathbf{u})$

[1](Rawlings and Mayne, 2009, pp.418-420)

LV control of distillation column



LV control of distillation column

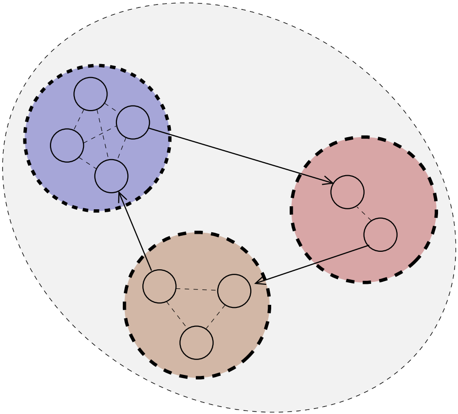


LV control of distillation column

Performance comparison

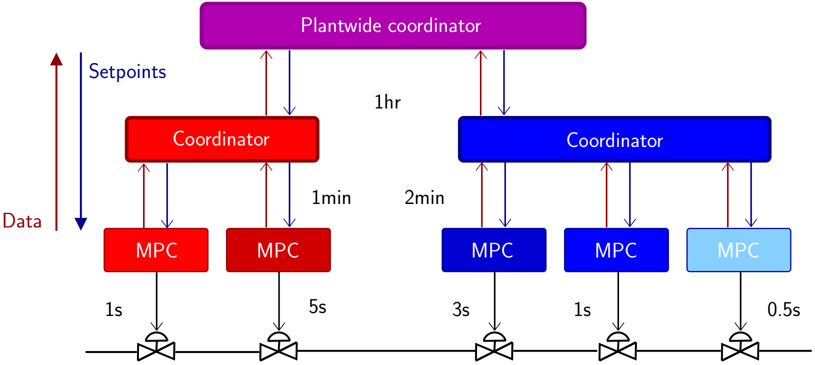
	Cost	Performance loss (%)
Centralized MPC	75.8	0
Cooperative MPC (10 iterates)	76.1	0.388
Cooperative MPC (1 iterate)	87.5	15.4
Noncooperative MPC	382	404
Decentralized MPC	364	380

Plantwide topology



- Plant subsystems can often be grouped spatially or dynamically
- Neighborhoods of subsystems naturally arise from topology

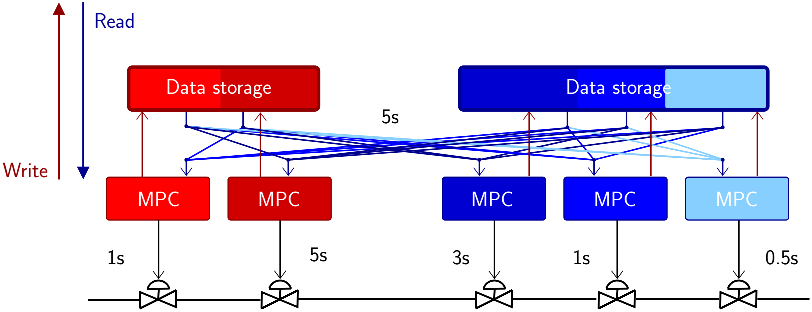
Traditional hierarchical MPC[2]



- Multiple dynamical time scales in plant
- Data and setpoints are exchanged on chosen scale
- Optimization performed at each layer

[2] Mesarović et al. (1970); Scattolini (2009)

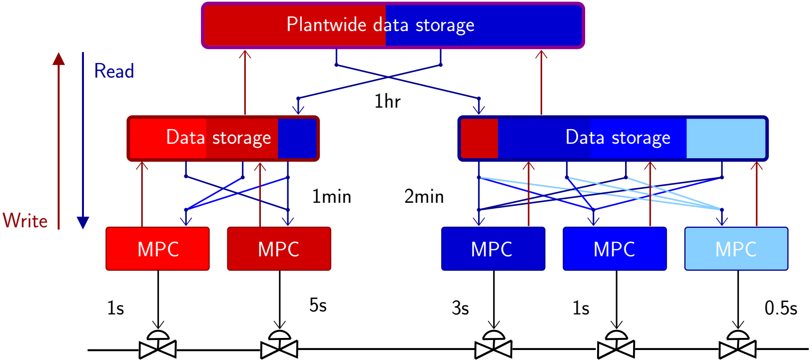
Cooperative MPC data exchange[3]



- All data exchanged plantwide
- Data exchange at each controller execution

[3]Venkat (2006); Stewart et al. (2010b)

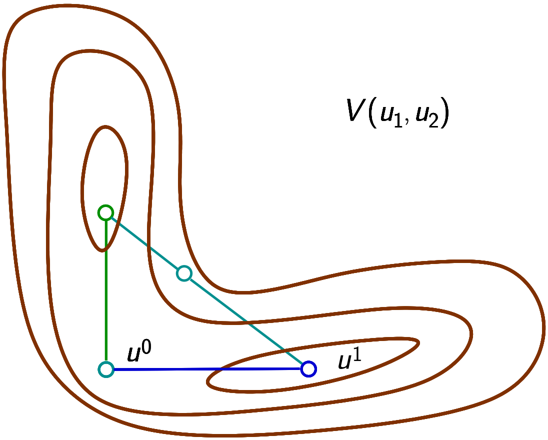
Cooperative hierarchical MPC[4]



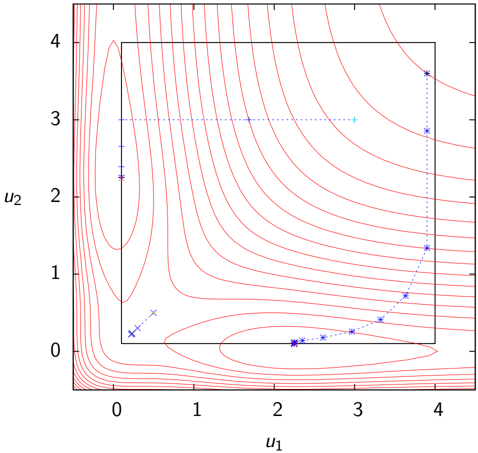
- Optimization at MPC layer only
- Only subset of data exchanged plantwide
- Data exchanged at chosen time scale

[4] Stewart et al. (2010a)

The challenge of nonlinear models



Distributed gradient projection - example



$$V(u_1, u_2) = e^{-2u_1} - 2e^{-u_1} + e^{-2u_2} - 2e^{-u_2}$$
$$+1.1\exp(-0.4((u_1 + 0.2)^2 + (u_2 + 0.2)^2))$$

Conclusions

Cooperative MPC theory maturing (Stewart et al., 2010b; Maestre et al., 2010)

- Satisfies hard input constraints
- Provides nominal stability for plants with even strongly interacting subsystems
- Retains closed-loop stability for early iteration termination
- Converges to Pareto optimal control in the limit of iteration
- Remains stable under perturbation from stable state estimator
- Avoids coordination layer

Future Outlook

Extensions required for practical implementation

- Can we treat nonlinear plant models? Qualified yes.
- Can we avoid coupled constraints? Qualified yes.
- Can we reduce the assumed complete communication? Yes.
- Can we accommodate time-scale separation? Yes.
- Can we nest layers within layers? Yes.

Personal observations of Tom

- He's old
  I took optimization from Tom when I was an undergraduate!
  I did research with Tom when I was an undergraduate!
  My son is now doing research with Tom as an undergraduate!
- He's wily
  I've played a lot of golf with Tom over the years at research meetings, and I want to make it clear he cheats!
  Tom was department chair when I joined the University of Texas as an assistant professor. He was a great mentor.
  Tom founded the Texas Modeling and Control Consortium in 1993;
  The Texas Wisconsin Modeling and Control Consortium in 1995;
  The Texas Wisconsin California Control Consortium in 2007.
- He's a lot of fun to be around

Further reading I

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